 http://dx.doi.org/10.18623/rvd.v23.6787

# SYNTHESIS OF MOBILE WALKING ROBOTS

*SÍNTESE DE ROBÔS MOTORES AMBULANTES*

Article received on: 1/23/2026
Article accepted on: 4/24/2026

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The authors declare that there is no conflict of interest

**Abstract**
A synthesis of a long kinematic chain with the ability to transform the chain from one species to another is shown. The method used is to divide the kinematic chain into functional units and to synthesize a space-periodic curve based on the spatial curve of the road. At the end, three results of solutions for this method of mobile walking robots, smooth road, hanging and flying are shown.

**Keywords:** Method. Walking Robots. Locomotion. Synthesis.

***Resumo***
*É apresentada uma síntese de uma longa cadeia cinemática com a capacidade de transformar a cadeia de um tipo para outro. O método utilizado consiste em dividir a cadeia cinemática em unidades funcionais e sintetizar uma curva espacial periódica com base na curva espacial da estrada. No final, são apresentados três resultados de soluções para este método em robôs mobilizes andantes: em estrada plana, suspensos e voadores.*

***Palavras-chave:*** *Método. Robôs Andantes. Locomoção. Síntese.*

## 1 INTRODUCTION

During locomotion, a person steps on one foot [1.2], then the other. This type of movement is considered more uniform, although there are many ways of locomotion with the human kinematic structure.

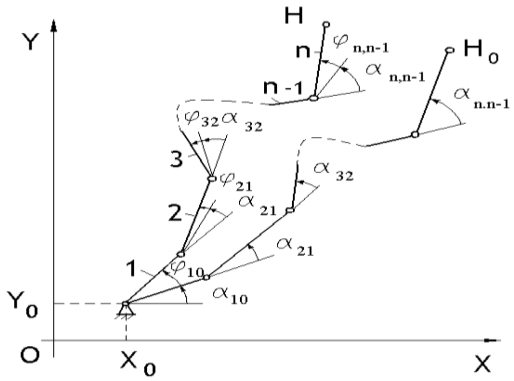
Human locomotion is described very well in the literature [3,4,5], as well as the synchronous activation of muscle groups for its implementation. There are moments when a person has stepped on only one foot and performs a movement [6,7]. At this moment, the human body obeys the laws of mechanics for the movement of a body with one fixed point, which is practically an open kinematic chain. However, there are also moments when the movement of the human body is in a position of two supports, that is, relative movement of a closed kinematic chain [8,9].

There are several ways to synthesize mechanisms for motion with one fixed point

A. The method proposed by Prof. Galabov [10] for structures and functions of guiding mechanisms with constant and variable gear ratios is a powerful method for synthesizing mechanisms in the field of locomotion. As he himself notes, the great similarity between the legs of walking robots and their movements and the movements of industrial robots and their structures gives grounds for unambiguity in the problems under consideration in terms of structure and kinematics.

**Figure 1**

*Conditionally open kinematic chain*




The coordinates of the characteristic point H of the working element of the manipulator (leg), i.e. the positional - zero transfer function, is determined by:

$$
\begin{aligned}
x_H &= x_0 + \sum_{i=1}^{n}\left[ l_i \cos\left( \sum_{k=1}^{i} \psi_{k,k-1} \right)\right] \\
y_H &= y_0 + \sum_{i=1}^{n}\left[ l_i \sin\left( \sum_{k=1}^{i} \psi_{k,k-1} \right)\right]
\end{aligned}
\qquad (1)
$$

After conversion, we get:

For three-link open kinematic chains (n=2) at $x_H = y_H = 0 \; and \; \alpha_{1,0} = \alpha_{2,1} = 0$ the above expressions reduce to:

$$
\begin{aligned}
\frac{d^{(m)}x_H}{d\varphi_{1,0}^{m}} &= l_1 \cos\left( \varphi_{1,0} + m\frac{\pi}{2} \right) + l_2\left(1-\varphi_{2,0}^{'(1)}\right)^{m} \cos\left[ \varphi_{1,0}\left(1-\varphi_{2,0}^{'(1)}\right) + m\frac{\pi}{2} \right] \\
\frac{d^{(m)}y_H}{d\varphi_{1,0}^{m}} &= l_1 \sin\left( \varphi_{1,0} + m\frac{\pi}{2} \right) + l_2\left(1-\varphi_{2,0}^{'(1)}\right)^{m} \sin\left[ \varphi_{1,0}\left(1-\varphi_{2,0}^{'(1)}\right) + m\frac{\pi}{2} \right]
\end{aligned}
\qquad (2)
$$

B. Let $e_1, e_2, e_3$ be an orthonormal basis in $E_3$defining the Cartesian system $Ox_1x_2x_3$.

The components of the tensor J in this basis, according to , are:

$$J_{11} = e_1.J.e_1 = \int_V (x_2^2 + x_3^2)dm = \int_v h^2\, dm \qquad (3)$$

$$J_{22} = \int_V (x_3^2 + x_1^2)dm \qquad (4)$$

$$J_{33} = \int_V (x_1^2 + x_2^2)\, dm \qquad (5)$$

$$J_{ij} = \int_V x_i.x_j dm, \qquad i \neq j \qquad (6)$$

With this approach, in addition to the Kinetic Momentum, the Kinetic Energy is also easily determined:

$$T = \frac{1}{2}\int_V v^2\, dm = \frac{1}{2}\omega \cdot J \cdot \omega\ . \qquad (7)$$

The introduction of the inertia tensor and its principal axes are the first and central moment in the derivation of the famous Euler equations of motion of an absolutely rigid body with one fixed point.

$$A\frac{dp}{dt} - (B - C)qr = M_{1'}^{ext} \qquad (8)$$

$$B\frac{dq}{dt} - (C - A)rp = M_{2'}^{ext} \qquad (9)$$

$$C\frac{dr}{dt} - (A - B)pq = M_{O}^{ext} \qquad (10)$$

An ellipsoid of inertia is also defined. With respect to the principal axes of inertia, the equation $Ax_{1'}^2 + Bx_{2'}^2 + Cx_{3'}^2 = 1$ is an equation of an ellipsoid. For an arbitrary positive definite and symmetric tensor *T*, the surface $r.T.r = 1$ is the so-called *tensor ellipsoid*.)

The specification of the ellipsoid of inertia $\zeta_J$ *uniquely* defines the tensor *J*. It is through the directions of its axes tha $\zeta_J$ defines its eigenvectors, and through the magnitudes of its semi-axes that it determines its eigenvalues.. In this way, the set of symmetric and positive definite tensors is identified with the set of all possible ellipsoids with center O, and each such tensor is associated with its tensor ellipsoid.

C. The model of Moreinis and Grichenko [11-16] studied the dynamic characteristics of healthy and injured people. They constructed a model of a human walking mechanism from a 9-link kinematic chain with 11 degrees of freedom. The authors wrote the differential equations of motion of the different parts of the mechanism in the form of Lagrange equations of the second kind:

$$\frac{d}{dt}\left(\frac{dT}{d\dot{q}_i}\right) - \frac{dT}{dq_i} + \frac{dV}{dq_i} = M_i \qquad (11)$$

The result is a system of five second-order nonlinear differential equations.

$$\begin{aligned}
&F_\varphi\left(\varphi,\dot{\varphi},\ddot{\varphi},\gamma,\dot{\gamma},\ddot{\gamma},x_1,\dot{x}_1,\ddot{x}_1,x_2,\dot{x}_2,\ddot{x}_2,\omega,\dot{\omega},\ddot{\omega},h,v\right) = M_\varphi\\
&F_\gamma\left(\varphi,\dot{\varphi},\ddot{\varphi},\gamma,\dot{\gamma},\ddot{\gamma},x_1,\dot{x}_1,\ddot{x}_1,x_2,\dot{x}_2,\ddot{x}_2,\omega,\dot{\omega},\ddot{\omega},h,v\right) = M_\gamma\\
&F_1\left(\varphi,\dot{\varphi},\ddot{\varphi},\gamma,\dot{\gamma},\ddot{\gamma},x_1,\dot{x}_1,\ddot{x}_1,x_2,\dot{x}_2,\ddot{x}_2,\omega,\dot{\omega},\ddot{\omega},h,v\right) = M_1 \qquad (12)\\
&F_2\left(\varphi,\dot{\varphi},\ddot{\varphi},\gamma,\dot{\gamma},\ddot{\gamma},x_1,\dot{x}_1,\ddot{x}_1,x_2,\dot{x}_2,\ddot{x}_2,\omega,\dot{\omega},\ddot{\omega},h,v\right) = M_2\\
&F\left(\omega,\dot{\omega},\ddot{\omega},h,v\right) = M_\omega
\end{aligned}$$

It is evident that this approach for multi-link structures is difficult to apply

D. In his works, Vukobratović [17-21] uses the synergetic method to solve problems related to walking mobile mechanisms. Vukobratović goes further by deriving a nonlinear differential equation for an arbitrary dynamic process.

$$\{M\} = [A]\left\{\ddot{\Xi}\right\} + [B]\left\{\dot{\Xi}^2\right\} + [C]\left\{\dot{\Xi}\dot{\Xi}\right\} + \{G\} \qquad (13)$$

The matrix coefficients $[A]$, $[B]$ and $[C]$, as well as the column matrix $\{G\}$ depend on the selected generalized coordinates of the dynamic process. Thus, dividing the movement of the kinematic chain into separate dynamics, it avoids the huge number of unknowns and achieves good results.

E. Recently proposed method [22-26], based on the division of mechanisms into functional units with a characteristic internal transfer function. In the most general case, the differential equation of the relationship between the output and input quantities for each functional unit can be written mathematically in the following form:

$$\mathbf{a_0 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_2 y = b_0 \frac{d^2x}{dt^2} + b_1 \frac{dx}{dt} + b_2 x} \qquad (14)$$

Thus, this method summarizes all previous methods and is based on the methods of automatic regulation and allows for a simplified version of the control of kinematic chains. With its help, long kinematic chains can be described.

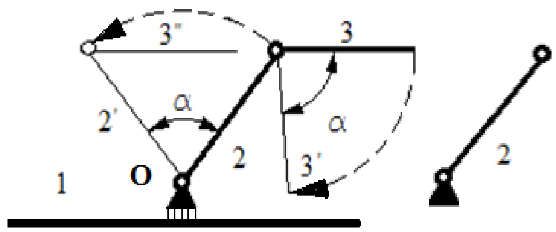
## 2 PRESENTATION

The recently published method [27-32] for mathematical modeling of the locomotion of the long kinematic chains will be used.

A. Starting from the basic scheme (formula) of the movement, the kinematic chain of the mobile mechanism is constructed.

**Figure 2**

*The place of the leg in the Basic formula of locomotion a) Basic formula of locomotion, b) the leg connecting the body 3 via a kinematic pair and the path 1 via a kinematic pair and a contact module.*




B. In an open kinematic chain, for a point of the i-th link relative to the absolute coordinate system, by multiplying the transformation matrices, we obtain:

$$\vec{r}_{H0} = T_1 T_2 T_3 \dots T_i \vec{r}_{H_i} \vec{r}_{H_i} \qquad (15)$$

where

$\vec{r}_{H0} = (x_{H0}, y_{H0}, z_{H0}, 1)^T, \vec{r}_{H_i} = \left(x_{H_i}, y_{H_i}, z_{H_i}, 1\right)^T$. T are two vectors connected by the multiplication of n-number of 4x4 matrices as many as there are links of the kinematic chain. When transforming the kinematic chain, some of the links of the chain are actually one link, due to hardening of the pairs between them. In this sense, the above equality takes the form: (16).

$$\vec{r}_{H0} = T_1 T_2 T_3 T_{4-7} T_8 T_9 \dots T_{k-k+t} \dots T_i \vec{r}_{H_i} \qquad (16)$$

where

$T_{4-7}$ and $T_{k-k+t}$ are transformation matrices of the sections of the kinematic chains (in this case two are shown, but there may be more), whose actuators only transform these sections, before the start of the movement, from one position to another, but do not participate in the considered movement of the entire kinematic system. Specifically, for (16), where there are two such sections, it can be written:

$$T_{4-7} = \prod_{i=4}^{7} T_i \qquad T_{k-k+t} = \prod_{i=k}^{t} T_i \qquad (17)$$

After substitution in (16) we obtain:

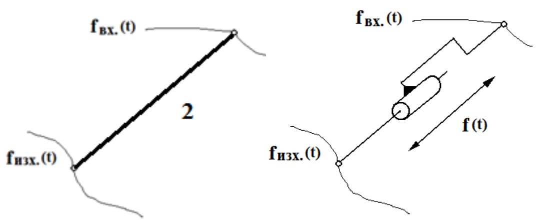
$$\vec{r}_{H0} = T_1 T_2 T_3 \prod_{i=4}^{7} T_i \, T_8 T_9 \ldots \prod_{i=k}^{t} T_i \ldots T_i \vec{r}_{H_i} \quad (18)$$

In (18) there will be as many products as there are transformation sections in the kinematic chain.

C. Long kinematic chains can be conditionally divided into sections (functional units), which when considered must take into account the equation of the relationship (14).

**Figure 3**

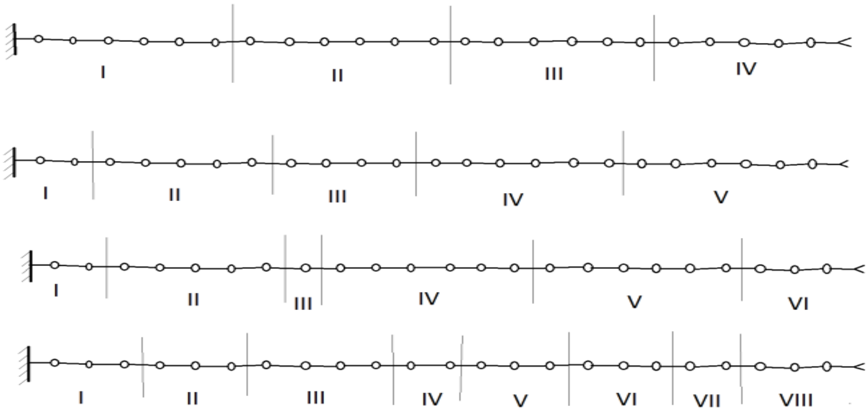
*Each functional unit changes the input motion through its internal motion into output motion*




These functional units can have autonomous low-level control. In this way, the design of the mechanics of the functional unit is parallel to its control.

**Figure 4**

*Open kinematic chain with 24 links and 23 kinematic pairs of the Vth class conditionally divided into sections in four variants.*




The open kinematic chain in Fig.4 has 24 links and 23 kinematic pairs of the Vth class and is a kinematic chain with high redundancy. In living nature, there are kinematic chains with significantly more links and many times higher redundancy, which function flawlessly.

If the control of the actuators is divided into sections in the same way as the kinematic chain, then in practice the transformation can take place so often - at an incremental level, that the movement is realized by all degrees of freedom without visibly noticing that separate sections of the kinematic chain are switched and driven.

D. Modeling the spatial periodic curve that the end points of the legs (ankle joints) must describe during locomotion is easily feasible using the method.

The basic formula for spatial periodic curves in this method is:

(19)

where

R – unbounded and non-orthogonal part, and L – bounded, orthogonal and normalized part of the linear combination of functions (19).

The physical meaning of this division is that the non-orthogonal part represents the equation of the path of motion of the walking mechanism (which is given in the assignment), and the orthogonal and normalized part is the periodic motion along it.

The exclusion of the R - curves from the above equation actually stops (excludes) the locomotion motion and only the relative motion of the future mechanism remains for study.

Since the main goal of the presentation is a synthesis of the locomotion mechanism, only the L - part is subjected to transformation, which is actually its relative motion.

Graphically presented, the L - part describes a closed loop in space, and therefore its projections are also closed loops. These are in practice the projections of the spatial periodic curve performed by the end point of the leg, i.e. the ankle joint.. It is necessary to clarify here that the two legs and the pelvic bridge between them are facilitated if they are divided into three functional units with their own transmission function. Further synthesis is a matter of engineering creativity.

E. The synthesis of a kinematic scheme for the implementation of a given spatial periodic curve is a multivariate task. The selection of a specific kinematic scheme is not only a matter of mathematical and engineering modeling, but it is also necessary to set heuristic restrictions for the choice.

After breaking the movement period into characteristic sections, the dependent and independent movements are separated by means of the partial derivatives in the Jacobi matrix. In practice, dependent movements use more than one unit to implement the movement, while independent ones use only one.

From the formulas (3,4,5,6) for the coefficients of the inertial tensors $J$ and the inertial ellipsoids, the load and the kinetic energy of the kinematic chain are expressed. When overloading certain sections, constructive changes are undertaken.

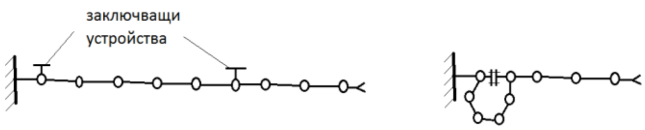
F. In variable locomotion systems with high redundancy. Open kinematic structures with redundancy can be transformed in two ways:

- The first way is with a simple removal of degrees of freedom by stiffening actuators. This can be practiced mainly in kinematic chains with low redundancy.
- The second way is when open kinematic chains with redundancy are transformed into open-closed or closed kinematic chains, with parts of these chains being closed in closed loops.

This second way can also include specialized locking devices with the help of which links located at a distance from each other in the kinematic chain at the moment of approach during movement can be locked to each other and form a new link or ignore the section of the kinematic chain locked between them by the general movement.

**Figure 5**

*Transformation of an open kinematic chain from nine degrees of freedom to five*




Closed kinematic chains with redundancy have wider possibilities for transformations. As with open chains, transformations can be carried out in two ways.

- The first is, as with open chains, by blocking the actuators.
- The second is by blocking not only active, but also inactive kinematic pairs. In this way, a kinematic chain with high redundancy can be transformed into different machines depending on the specific needs.

From the formula for degrees of freedom at h = 1, we get r = 1, n = 4, p5 = 4, i.e. this is a four-link. A four-link is a mechanism with one degree of freedom. Variability generally takes away degrees of freedom from kinematic chains and in this case would turn the four-link into a triangular, that is, truss element..

At r = 4 and i = 2, one of the possible variants of these structures is shown in Figure 6

**Figure 6**

*Transformation of one mechanism with a closed structure to two different mechanisms*

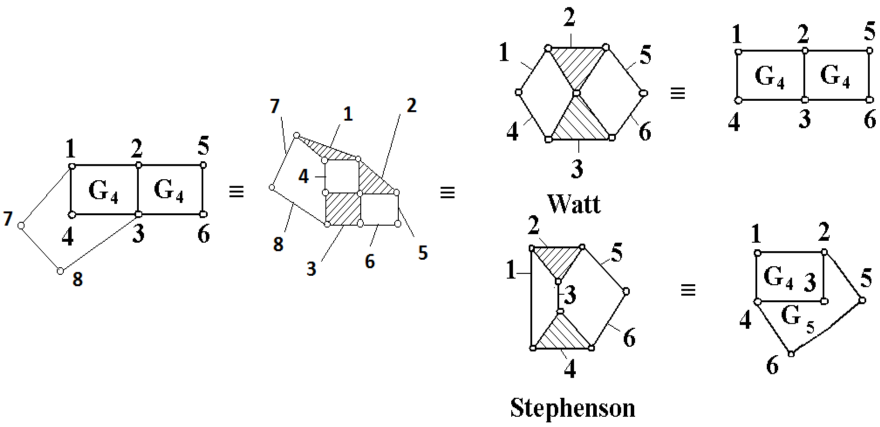



Figure 6 clearly shows that the closed kinematic structure with two degrees of freedom can be transformed by blocking one degree of freedom and the mechanism becomes a Watt mechanism, and by blocking the other degree of freedom the mechanism becomes a Stephanson mechanism.

## 3 CONCLUSIONS

The new proposed method, based on the division of mechanisms into functional units with a characteristic internal transmission function, is a very powerful operator in the process of designing complex complexes with long kinematic chains. It is based on previous experience in this area and experience in the field of automatic regulation and allows for a smooth connection between mechanics and control.

The presented method complements and illustrates the achievements in the field of designing long kinematic chains and predisposes to modular control.

Some results designed using conditional units:

A. Analytical and graphical view of the spatial periodic curve of one leg of a robot performing locomotion by hanging.

$a := 5$

$b := 3$

$c := 2$

$$d := 4$$

$$y(x) := \left[ a \cdot \left( x \cdot 4 - c \cdot \sin\left( x \cdot d + \frac{b}{2} \right) \right) + c \cdot \left( 1 - \pi \cdot \cos(x \cdot d \cdot 2 + b) \right) \right]$$

$$z(x) := a \cdot \left( 1 - \cos\left( x \cdot d + \frac{b}{2} \right) \right) + \pi \cdot \sin\left( d \cdot x + \frac{b}{2} \right) \qquad (20)$$

$$f(x) := \left[ a \cdot \left( 4 - c \cdot \sin\left( x \cdot d + \frac{b}{2} \right) \right) + c \cdot \left( 1 - \pi \cdot \cos(x \cdot d \cdot 2 + b) \right) \right]$$

$$n(x) := a \cdot \left( 1 - \cos\left( x \cdot d + \frac{b}{2} \right) \right) + \pi \cdot \sin\left( d \cdot x + \frac{b}{2} \right)$$

**Figure 7**

*Spatial periodic curve projections*

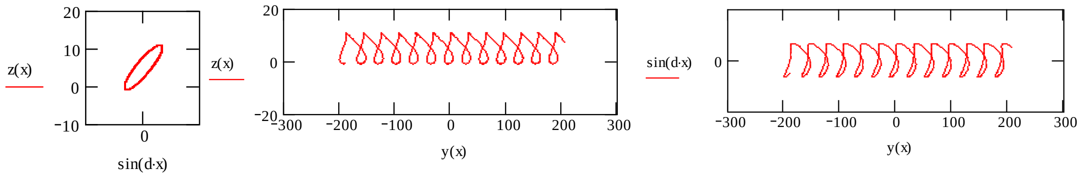
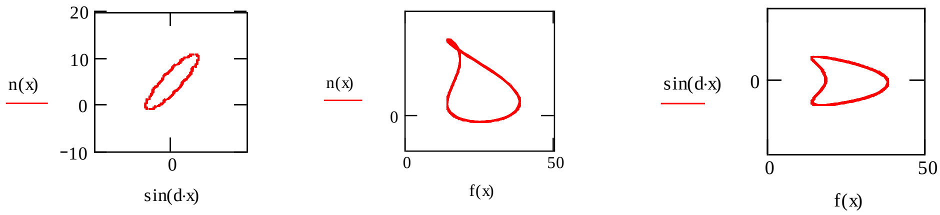



**Figure 8**

*Projections of relative movements*




B. Analytical and graphical representation of the spatial periodic curve of one leg of a robot performing locomotion on hard terrain, but with the ability to overcome higher obstacles.

$$a := 10$$

$$b := 2$$

$$c := 5$$

$$d := 2$$

$$y(x) := [\, a \cdot (x \cdot 4 - c \cdot \sin(d \cdot x + b)) + c \cdot 4 \cdot [\, \cos((d \cdot x \cdot 2 + b))\, ]\, ]$$

$$z(x) := a \cdot (1 - \cos(d \cdot x + b)) \quad f(x) := [\, a \cdot (4 - c \cdot \sin(d \cdot x + b)) + c \cdot 4 \cdot [\, \cos((d \cdot x \cdot 2 + b))\, ]\, ] \tag{21}$$

$$v(x) := \sin(d \cdot x)$$

$$n(x) := a \cdot (1 - \cos(d \cdot x + b))$$

**Figure 9**

*Spatial periodic curve projections*

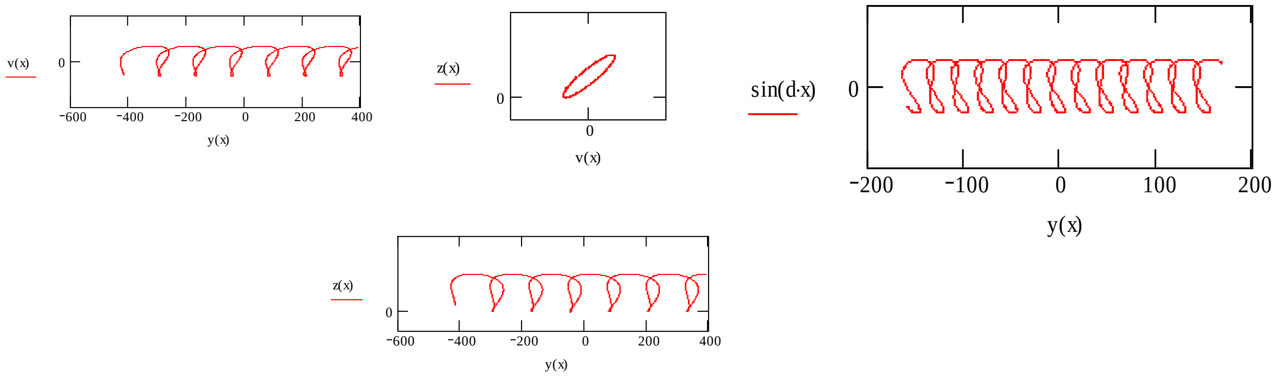
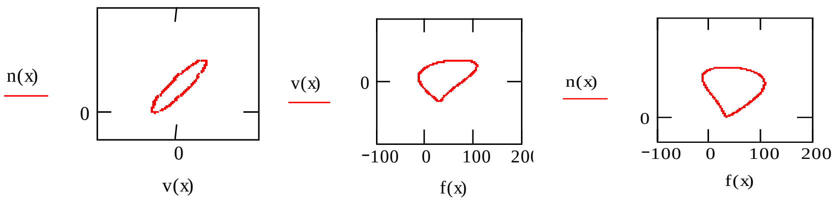



**Figure 10**

*Projections of relative movements*




C. Analytical and graphical representation of a spatial periodic curve of one endpoint of a robot performing locomotion by flying.

$$a := 4$$

$$b := 2$$

$$c := 2$$

$$d := 4$$

$$y(x) := \left[ a \cdot \left( x \cdot 4 - c \cdot \sin\left( d \cdot x + \frac{b}{2} \right) \right) + c \cdot [\, 4 \cdot \cos((d \cdot x \cdot 2 + b))\, ] \right]$$

$$z(x) := a \cdot \left( 1 - \cos\left( d \cdot x + \frac{b}{2} \right) \right) \qquad (22)$$

$$f(x) := a \cdot \left( 4 - c \cdot \sin\left( d \cdot x + \frac{b}{2} \right) \right) + c \cdot [\, 4 \cdot \cos((d \cdot x \cdot 2 + b))\, ]$$

**Figure 11**

*Spatial periodic curve projections*

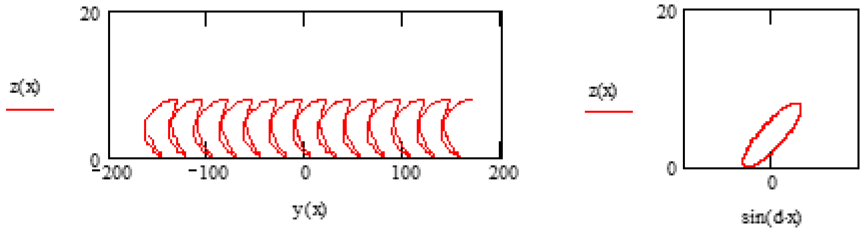



**Figure 12**

*Projections of relative movements*

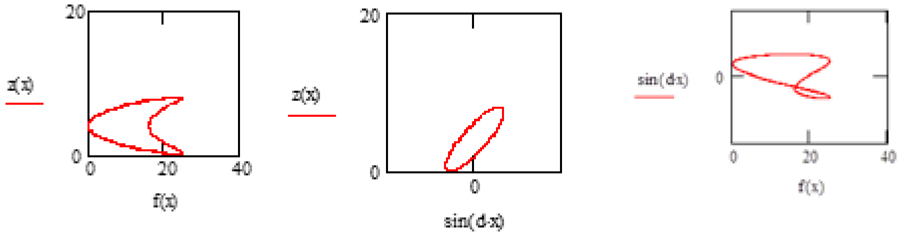
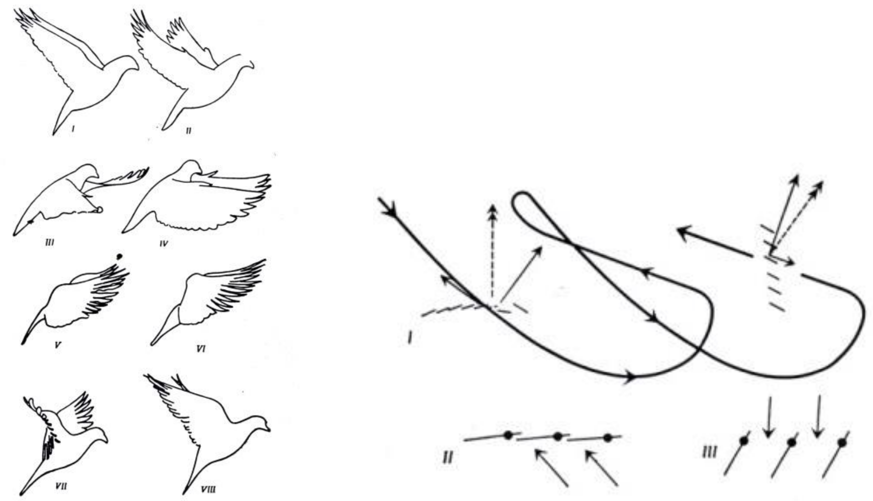


**Figure 13**

*The graph was taken down by ornithologists for the movement of a pigeon's wing for a specific flight (gait)*



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