http://dx.doi.org/10.18623/rvd.v23.n4.4471

INCREASING THE ACCURACY OF THE ANTI-AIRCRAFT MISSILE HOMING SYSTEM BY CHANGING THE PROPORTIONALTY COEFFICIENT OF THE HOMING LAW

AUMENTO DA PRECISÃO DO SISTEMA DE GUIAMENTO DE MÍSSEIS ANTIAÉREOS POR MEIO DA ALTERAÇÃO DO COEFICIENTE DE PROPORCIONALIDADE DA LEI DE GUIAMENTO

Article received on: 10/16/2025
Article accepted on: 1/16/2026

Nguyen Ngoc Tuan*
*Le Quy Don Technical University, Hanoi, Viet Nam
Orcid: https://orcid.org/0009-0000-1572-7702
tuannn@lqdtu.edu.vn

Do Quang Thong*
*Le Quy Don Technical University, Hanoi, Viet Nam
Orcid: https://orcid.org/0009-0004-1275-6662
thongdq@lqdtu.edu.vn

The authors declare that there is no conflict of interest

Abstract
This paper presents a method for improving the accuracy of an anti-aircraft missile homing system when engaging highly maneuvering targets by adjusting the proportionality coefficient of the guidance law. First, a mathematical model and the synthesis of the anti-aircraft missile homing system in the vertical plane are presented. Then, the dependence of the accuracy of the synthesized system on changes in the aerodynamic coefficients and the speed of the missile and the moment of the target's one-sided maneuver is investigated. A law for varying the proportionality coefficient and an algorithm for determining its parameters are proposed. To reduce the impact of interference on the system's accuracy, a Kalman filter is proposed. Simulation of the synthesized system is carried out using the Simulink environment. As a result, a high-accuracy anti-aircraft missile homing system is obtained.

Keywords: System Synthesis. Missile. Missile Homing System. Proportional Guidance Method.

Resumo
Este artigo apresenta um método para melhorar a precisão de um sistema de guiamento de mísseis antiaéreos ao engajar alvos altamente manobráveis, por meio do ajuste do coeficiente de proporcionalidade da lei de guiamento. Inicialmente, são apresentados o modelo matemático e a síntese do sistema de guiamento do míssil antiaéreo no plano vertical. Em seguida, investiga-se a dependência da precisão do sistema sintetizado em relação às variações dos coeficientes aerodinâmicos, da velocidade do míssil e do instante da manobra unilateral do alvo. Propõem-se uma lei de variação do coeficiente de proporcionalidade e um algoritmo para a determinação de seus parâmetros. Para reduzir a influência de interferências na precisão do sistema, é proposto o uso de um filtro de Kalman. A simulação do sistema sintetizado é realizada no ambiente Simulink. Como resultado, obtém-se um sistema de guiamento de mísseis antiaéreos de alta precisão.

Palavras-chave: *Síntese de Sistemas. Míssil. Sistema de Guiamento de Mísseis. Método de Guiamento Proporcional.*

1 INTRODUCTION

Surface-to-air missiles equipped with homing systems are increasingly widely used due to their ability to maintain high guidance accuracy regardless of engagement range and to operate in a "fire-and-forget" mode. Improving guidance accuracy remains a primary objective in missile system design. In contrast, the survivability of aerial targets is continuously enhanced, particularly through improved maneuverability. Modern aircraft equipped with thrust vector control exhibit high maneuvering capabilities. Typical maneuvers include unilateral maneuvering, horizontal weaving, and spatial barrel rolls [1]. Such maneuvers increase missile guidance errors, making accurate guidance against highly maneuverable targets an important challenge.

Several approaches have been proposed to address this problem. A biased proportional navigation method was introduced in [2], while proportional navigation with lead and instantaneous miss distance homing methods were proposed in [3, 4]. However, these methods involve complex technical implementation, requiring the estimation of the projection of the line-of-sight angular rate onto the antenna coordinate system ω_a and the ω_T component that compensates for target maneuvering. Accurate onboard estimation of ω_T is difficult, as it requires knowledge of the target's normal acceleration [4].

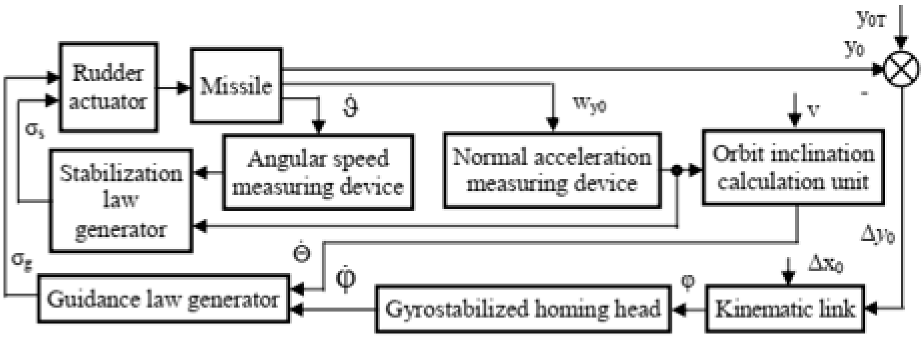
This paper investigates a method for improving guidance accuracy against targets performing a unilateral maneuver, such as disengagement upon detection of a surface-to-air missile launch. Other maneuver types are beyond the scope of this study and will be considered in future work. The synthesis of a surface-to-air missile homing system is first presented to demonstrate the effectiveness of the proposed approach. In the missile homing system, the target coordinator (homing head) and the control organs (rudder drive) are located close to each other. If the axes of rotation of the rudder, the antenna of the homing head, the planes of the rudder and wing are parallel, then we can assume that the two guidance planes are independent. The synthesis of the homing system in two planes is similar [5]. Below, we will present the synthesis of the missile homing system in the vertical plane.

2 MISSILE HOMING SYSTEM SYNTHESIS

The functional block diagram of the surface-to-air missile homing system (SAMHS) in the vertical plane is shown in Fig. 1, while the relative geometry between the missile and the target is illustrated in Fig. 2 [5–7].

Figure 1

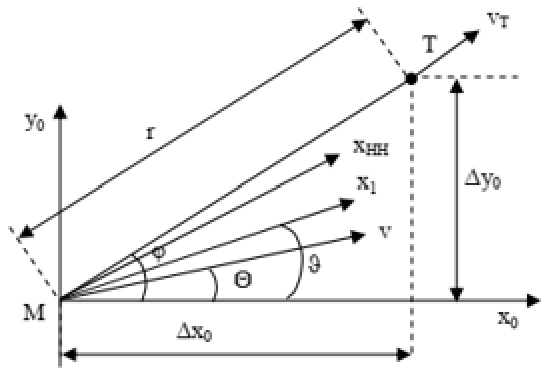
The functional diagram of the missile homing system



In Figs. 1 and 2:
y_0 [m] - missile altitude;
y_{0T} [m] - target altitude;
Δy_0 [m] - difference between missile and target altitudes;

Figure 2

The relative position of the missile and the target

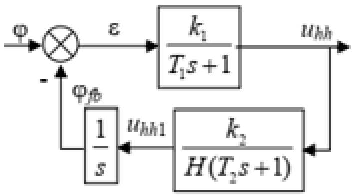


Δx_0 [m] - difference of the missile and target coordinates along the horizontal axis x_0;
v [m/s] - missile speed;
w_{y0} [m/s^2]- missile normal acceleration;
φ [rad] - inclination angle of the line-of-sight (missile–target);

Θ [deg] - inclination angle of the missile velocity vector;
ϑ [deg] - missile pitch angle;
σ_s [V] - stabilization signal;
σ_g [V] - guidance signal;
M – missile;
T- target;
r [m] - slant range between the missile and the target;
x_{HH} - optical axis of the homing head;
x_1 - longitudinal axis of the missile;
v_T [m/s]- target speed.

Figure 3

The structural diagram of the gyro-stabilized homing head



The structural diagram of the gyro-stabilized homing head is shown in Fig. 3 [8]. Here, k_1, T_1 are the gain and time constant of the amplifier and filter; k_2, T_2 are the conversion coefficient and time constant of the moment (torque) generator; H is the gyroscope's kinetic moment; u_{hh} is the output signal of the gyro-stabilized homing head.

In the vertical plane, the mathematical model of a missile with fixed fins, for small rudder rotation angle (δ) and small angle of attack (α), in the form of transfer functions and first-order differential equations, is as follows [4, 5-9]:

$$\frac{\vartheta(s)}{\delta(s)} = -\frac{a_{13}s + a_{13}a_{42}}{s[s^2 + (a_{11} + a_{42})s + a_{12} + a_{11}a_{42}]}; \qquad (1)$$

$$\frac{\Theta(s)}{\delta(s)} = -\frac{a_{13}a_{42}}{s[s^2 + (a_{11} + a_{42})s + a_{12} + a_{11}a_{42}]}; \qquad (2)$$

$$\frac{\dot{\Theta}(s)}{\dot{\vartheta}(s)} = \frac{a_{42}}{s + a_{42}}; \qquad (3)$$

$$\frac{\alpha(s)}{\delta(s)} = -\frac{a_{13}}{s^2 + (a_{11} + a_{42})s + a_{12} + a_{11}a_{42}}; \qquad (4)$$

$$\dot{\Theta}(s) = \frac{w_{y0}(s)}{v}; \qquad (5)$$

$$\begin{cases} \dot{\omega}_{z1} = -a_{11}\omega_{z1} - a_{12}\alpha - a_{13}\delta; \\ \dot{\vartheta} = \omega_{z1}; \\ \dot{\Theta} = a_{42}\alpha; \\ \alpha = \vartheta - \Theta; \\ w_{y0} = v a_{42}\alpha; \end{cases} \qquad (6)$$

where:

δ - rudder angle [deg];

ω_{z1}-missile rotation rate about the o_{z1} axis [deg/s];

α-attack angle [degree];

a_{11}-natural damping coefficient [1/s];

a_{12}-the coefficient of windage [1/s²];

a_{13}-rudder efficiency coefficient [1/s²];

a_{42}-normal-force coefficient [1/s].

We apply the mathematical model of the rudder actuator, angular-rate sensor, and accelerometer in the form of an oscillatory element [3, 5, 8]. The general form of their transfer function is:

$$W(s) = \frac{Y(s)}{X(s)} = \frac{k}{T^2 s^2 + 2\xi T s + 1}, \qquad (7)$$

where:

x-input signal;

y-output signal;

k-conversion (gain) coefficient;

ξ-damping coefficient;

T-time constant.

Transform equation (7) into differential equations:

$$\begin{cases} \dot{y}_1 = \frac{k}{T^2} x - \frac{1}{T^2} y - \frac{2\xi}{T} y_1; \\ \dot{y} = y_1. \end{cases} \quad (8)$$

For the angular-rate sensor: $x=\omega_{z1}$ [deg/s]; $k=k_{as}$ [V/(deg/s)]; $\xi=\xi_{as}$; $T=T_{as}$ [s]; the output signal is $y=u_{as}$ [V].

For the accelerometer: $x=w_{y0}$ [m/s²]; $k=k_{ak}$ [V/(m/s²)]; $\xi=\xi_{ak}$; $T=T_{ak}$ [s]; the output signal is $y=u_{ak}$ [V].

For the rudder actuator: $x=u_r$ [V]; $k=k_r$ [deg/V]; $\xi=\xi_r$; $T=T_r$ [s]; the output signal is $y=\delta$ [deg].

The signals u_r, σ_s, and σ_g are determined as follows [5]:

$$\begin{cases} u_r = \sigma_s - \sigma_g; \\ \sigma_s = k_w u_{ak} + k_{\omega z1} u_{as}; \\ \sigma_g = k(k_{pr} u_{hh} - u_{ak} / k_{ak} v) \end{cases} \quad (9)$$

where k_w, $k_{\omega z1}$-gain coefficients in the feedback loops of the normal-acceleration stabilization system (stabilization-law coefficients); k, k_{pr}-gain coefficient and proportionality coefficient of the proportional guidance method [5].

Kinematic equations of missile motion:

$$\begin{cases} \dot{x}_0 = v \cos\Theta; \\ \dot{y}_0 = v \sin\Theta; \end{cases} \quad (10)$$

Kinematic equations of target motion:

$$\begin{cases} w_{y0T} = \begin{bmatrix} 0;\ npu\ t \le t_{м} \\ const; npu\ t > t_{м}; \end{bmatrix} \\ \dot{\Theta}_T = \dfrac{w_{y0T}}{v_T}; \\ \dot{x}_{0T} = v_T \cos \Theta_T; \\ \dot{y}_{0T} = v_T \sin \Theta_T; \end{cases} \qquad (11)$$

where:

w_{y0T}-target normal acceleration [m/s²];

t-guidance time [s];

t_m-time of target maneuver relative to the start of the homing process [s];

Θ_T-target trajectory inclination angle [rad];

x_{0T} [m],

y_{0T} [m]-target coordinates along the axes of the Earth's coordinate system in the vertical plane.

Equations of relationships between the missile and target coordinates (see Fig. 2):

$$\begin{cases} \Delta x_0 = x_{0T} - x_0; \\ \Delta y_0 = y_{0T} - y_0; \\ r = \sqrt{\Delta x_0^2 + \Delta y_0^2}; \\ \varphi = arctg \dfrac{\Delta y_0}{\Delta x_0}. \end{cases} \qquad (12)$$

By converting the transfer functions in Fig.3 into first-order differential equations, we obtain:

$$\begin{cases} \varepsilon = \varphi - \varphi_{fb}; \\ \dot{u}_{hh} = \dfrac{k_1}{T_1}\varepsilon - \dfrac{u_{hh}}{T_1}; \\ \dot{u}_{hh1} = \dfrac{k_2}{HT_2}u_{hh} - \dfrac{u_{hh1}}{T_2} \\ \dot{\varphi}_{fb} = u_{hh1}. \end{cases} \qquad (13)$$

The missile homing system employing the proportional navigation method contains a gyro-stabilized homing head. In its design, it is necessary to ensure the stability of the gyro-stabilized homing head. Applying the Hurwitz stability criterion to the systems in Fig.3, we obtain:

$$0 < k_1 < (\frac{1}{T_1} + \frac{1}{T_2}) \frac{H}{k_2}. \qquad (14)$$

When synthesizing the missile homing system using the parametric optimization method, it is necessary to transform systems (6), (8)-(12), and (13) into a system of 17 first-order differential equations and integrate them. The task of synthesizing the missile homing system is to determine the optimal values of the coefficients $k_{\omega z1opt}$, k_{wopt}, k_{opt}, k_{propt}) that provide the smallest guidance error. For simplicity of calculation it is assumed that the target moves in a straight line with constant speed; the speed of the missile does not change; the guidance error is defined as the distance between the missile and the target at the end of the homing process [5].

The initial target coordinates are assumed to be x_{0t}=5000 m and y_{0t}=3000 m, with head-on firing (Θ_t=180 degree).

It is assumed that [5] a_{11}=1.2 s-1; a_{12}=20 s-2; a_{13}=28 s-2; a_{42}=1.4 s-1; v=1300 m/s; k_r=1 deg/V; ξ_r=0.6; T_r=0.05 s; δ_{max}=±20 deg; k_{as}=1 V/(deg/s); ξ_{as}=0.7; T_{as}=0.04 s; k_{ak}=1 V/(m/s2); ξ_{ak}=0.6; T_{ak}=0.03 s; It is assumed that the parameters of the gyro-stabilized homing head are: k_2/H=0.1; T_1=0.1 s; T_2=0.05 s; k_1=150, satisfying (14).

The algorithm for synthesizing the missile homing system is summarized as follows.

Step 1. The parameter $k_{\omega z1}$ is scanned from kωz1min to $k_{\omega z1max}$ with a sufficiently small step size $dk_{\omega z1}$. For each value of $k_{\omega z1}$, the parameter k_w is scanned from k_{wmin} to k_{wmax} with a sufficiently small step size dk_w. For each pair ($k_{\omega z1}$, k_w), the closed-loop transfer function of the normal acceleration stabilization system is constructed using the Control System Toolbox (MATLAB). The stability of the system is then evaluated according to the Hurwitz criterion, and the stability margin in terms of the gain margin Pm is determined. If the gain margin Pm exceeds 5 dB, the parameter k is scanned from k_{min} to k_{max} with a sufficiently small step size dk. For each combination of $k_{\omega z1}$, k_w, and k, the parameter k_{pr} is scanned with a sufficiently small step size dk_{pr}.

Step 2. For each parameter set ($k_{\omega z1}$, k_w, k, k_{pr}), the system of 17 differential equations describing the missile homing system is numerically integrated over the entire guidance process. The guidance error Δr is evaluated, and a row of the matrix Kq is formed containing the values ($k_{\omega z1}$, k_w, k, k_{pr}, Pm, Δr). In addition, the synthesis results show that in order for the self-guidance system to be able to shoot targets with different initial coordinates with high accuracy, when synthesizing, it is necessary to choose a missile trajectory that is as straight as possible or has only a slight upward curve. To achieve that, it is necessary to ensure that the difference (y_0(t)-y_0(t-T_0)) is not less than 1.6 m. Therefore, after each integration step, it is necessary to check that if the difference (y_0(t)-y_0(t-T_0)) is less than 1.6 m, then the integration for this parameter set ($k_{\omega z1}$, k_w, k, k_{pr}) needs to be terminated.

Step 3. The five rows of matrix Kq corresponding to the smallest values of the guidance error Δr are selected. For each selected row, the refined parameter ranges are defined as $k_{\omega z1min}=k_{\omega z1}-dk_{\omega z1}$; $k_{\omega z1max}=k_{\omega z1}+dk_{\omega z1}$; $k_{wmin}=k_w-dk_w$; $k_{wmax}=k_w+dk_w$; $k_{min}=k-dk$: $k_{max}=k+dk$; $k_{prmin}=k_{pr}-dk_{pr}$; $k_{prmax}=k_{pr}+dk_{pr}$.

Step 4. Steps 1-3 are repeated using significantly smaller scanning steps $dk_{\omega z11} \ll dk_{\omega z1}$, $dk_{w1} \ll dk_w$, $dk_1 \ll dk$, and $dk_{pr1} \ll dk_{pr}$. At this stage, a new matrix Kq1 is generated in the same manner as matrix Kq.

Step 5. The row of matrix Kq1 corresponding to the minimum guidance error Δr is identified. The optimal parameter set (($k_{\omega z1opt}$, k_{wopt}, k_{opt}, k_{propt}) is thus obtained.

Having performed the synthesis, we get $k_{\omega z1opt}$=0,16; k_{wopt}=0,006; k_{opt}=7; k_{propt}=4. Let's call this system System 1.

2 INVESTIGATION OF THE INFLUENCE OF VARIATIONS IN AERODYNAMIC COEFFICIENTS, MISSILE SPEED AND HOMING HEAD SYSTEMATIC ERROR ON THE ACCYRACY OF THE SYNTHEZED MISSILE HOMING SYSTEM

The mathematical model of the missile described by transfer functions (1)-(5) is approximate. Its aerodynamic coefficients a_{11}, a_{12}, a_{13}, a_{42} are determined with some error. In addition, during flight, changes in mass, missile speed, and air density cause variations in aerodynamic coefficients. Therefore, during flight, the actual values of these coefficients differ from the calculated values a_{11c}, a_{12c}, a_{13c}, a_{42c}. Consequently, it is necessary to investigate the influence of variations in the aerodynamic coefficients and the missile speed on the accuracy of the missile homing system. It is assumed that the values of the aerodynamic coefficients and speed differ from the calculated ones by ±20%. We investigate the accuracy of system 1 when firing at nonmaneuvering targets with different initial coordinate differences between the missile and the target. It is assumed that the target moves rectilinearly at a constant speed of 800 m/s, with Θ_t=180°. The guidance errors are listed in Table 1.

Table 1

Guidance error of the synthesized system under variations in aerodynamic coefficients and missile speed

Initial coordinate difference	Δx_0, m	5000	5000	7000	7000	7000	12000	12000	15000
	Δy_0, m	4000	5000	4000	5000	6000	4000	7000	5000
The value of aerodynamic coefficients	Missile speed	Guidance error, m							
a_{42c}; a_{12c} a_{11c}; a_{13c}	v_c	1.27	1.69	0.82	0.91	0.3	0.33	0.16	0.11
1.2a_{42c}; 1.2a_{12c} 1.2a_{11c}; 1.2a_{13c}	0.8v_c v_c 1.2v_c	0.29 1.29 **8.25**	0.45 0.5 **4.6**	0.21 0.5 **1.1**	0.69 0.73 0.33	0.57 0.37 0.84	0.11 0.09 **1.07**	0.05 0.32 1	0.52 0.39 0.83
1.2a_{42c}; 1.2a_{12c} 1.2a_{11c}; 0.8a_{13c}	0.8v_c v_c 1.2v_c	0.75 0.68 1.11	0.19 0.3 1.4	0.97 0.66 0.38	0.52 0.77 0.58	0.39 0.57 0.36	0.5 0.79 0.59	0.34 0.59 0.21	0.78 **0.99** 0.52
…	…								
0.8a_{42c}; 1.2a_{12c} 1.2a_{11c}; 0.8a_{13c}	0.8v_c v_c 1.2v_c	**2.97** 4.42 **5.76**	**2.28** 3.24 **3.87**	**1.19** 2.52 **3.13**	**1.6** 2.48 **2.96**	**1.64** 2.19 **2.59**	0.65 0.98 **1.15**	0.67 1.47 **1.61**	0.51 0.78 **1.07**
0.8a_{42c}; 1.2a_{12c} 0.8a_{11c}; 0.8a_{13c}	0.8v_c v_c 1.2v_c	**2.83** **3.85** **5.62**	**2.05** **2.9** **2.98**	**1.31** **2.39** **2.81**	**1.7** **2.33** **2.76**	**1.5** **1.94** **2.17**	0.23 **1.21** **1.4**	0.83 **1.24** **1.39**	0.45 0.64 0.89
0.8a_{42c}; 0.8a_{12c} 0.8a_{11c}; 0.8a_{13c}	0.8v_c v_c 1.2v_c	1 0.61 **1.8**	0.42 **1.03** **1.23**	1 0.79 0.61	0.75 0.13 0.61	0.68 0.45 **1.01**	0.5 0.45 **1**	0.44 0.12 0.6	0.92 0.86 0.28

As shown in Table 1, when the missile aerodynamic coefficients vary within ±20%, the guidance error can reach up to 8.25 m when engaging targets at short range, whereas for long-range engagements the guidance error increases only slightly. An increase in missile velocity intensifies the effect of variations in aerodynamic coefficients on the guidance error, while a reduction in missile velocity mitigates this effect.

Therefore, for aerodynamic coefficient variations within ±20%, a longer guidance time results in a smaller influence of these variations on the guidance error, whereas a shorter guidance time leads to a more pronounced influence.

3 INCREASING THE ACCURACY OF THE SYTHEZED SYSTEM WHEN FIRING AT MANEUVERABLE TARGETS BY CHANGING THE PROPORTIONALITY COEFFICIENT

First, the accuracy of the synthesized system is investigated for engagements against maneuvering targets. The target is assumed to move at a constant velocity of 800 m/s with initial coordinate differences between the missile and the target of Δx_0=15000 m and Δy_0=5000 m, and an initial target heading angle of Θ_t = 180°. At time instants t_m = 4 s, t_m = 5 s, t_m = 5.4 s, 5.6 s, … after the start of homing, the target performs a maneuver with an acceleration of −70 m/s². The guidance error Δr is presented in Table 2.

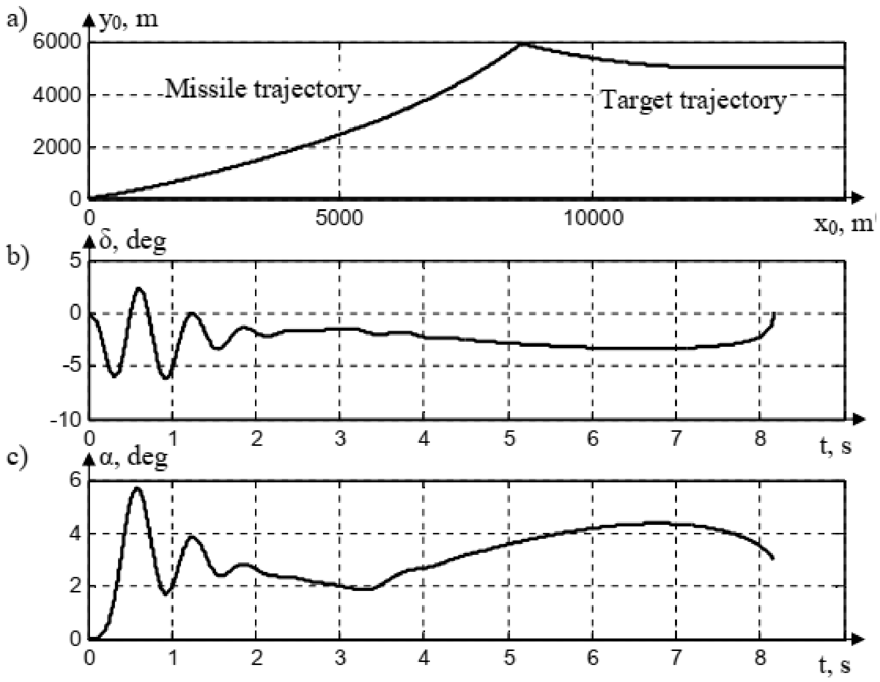
Table 2

Error in aiming at maneuvering targets

Δx_0=15000 m; Δy_0=5000 m; v_t=800 m/s; Θ_t=180 deg; w_t=-70 m/s^2														
t_m, c	4	5	5.4	5.6	5.8	6	6.2	6.4	6.6	6.8	7	7.2	7.4	7.6
k_{pr}	$k_{pr}=k_{propt}=4$													
Δr, m	0.81	0.86	0.86	0.97	0.59	0.8	2.15	3.56	1.8	3.69	8.69	8.69	4.85	1.4
k_{pr}	4	4	4	4	4	4	3.5	3.5	3.8	4.6	8	14	14.5	9
	0.81	0.86	0.86	0.97	0.6	0.8	0.85	0.9	0.66	0.36	0.65	1	1.05	1.16

Figure 4

Target and missile trajectories (a), and the missile's rudder angle (b) and angle of attack (c) over time



For t_m=3.2 s, the simulation results of the homing system are shown in Fig. 4. The missile and target trajectories are presented in Fig.4,a, the time history of the control surface deflection angle is shown in Fig.4,b, and the angle-of-attack variation is depicted in Fig.4,c. As can be seen from Fig. 4, the missile trajectory is slightly curved upward, as intended by the design; the maximum values of both the rudder angle and the angle of attack do not exceed 20°. Therefore, the use of linearized missile equations of motion is justified.

As shown in Table 2, when a fixed coefficient $k_{pr}=k_{propt}$ is used, the guidance error can reach up to 14.5 m during target maneuvering. In each maneuvering scenario, selecting an appropriate value of k_{pr} significantly reduces the guidance error. Therefore, the problem is to determine the onset of the target maneuver and to select the corresponding coefficient k_{pr}. The output signal of the homing seeker is used to detect the target maneuver onset, and a law for varying k_{pr} is formulated. To this end, finite-difference signals are constructed as follows:

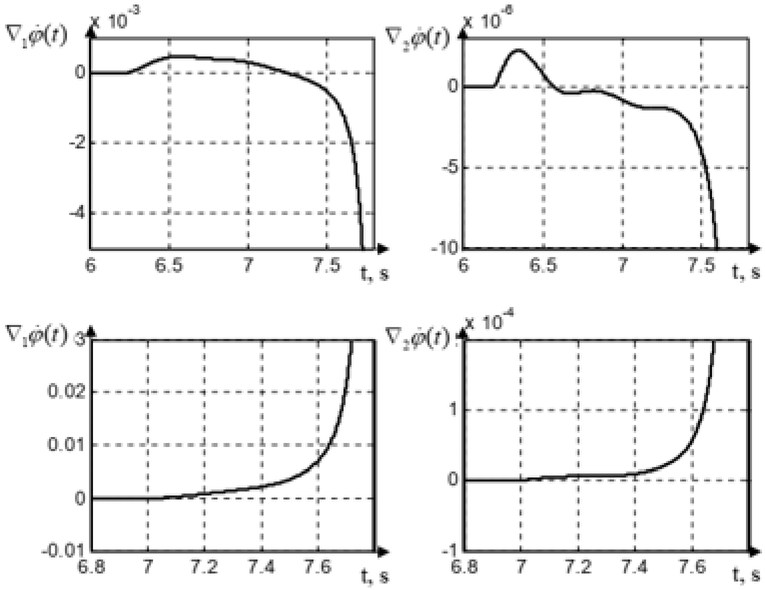
$$\nabla_1\dot{\varphi}(t)=\dot{\varphi}(t)-\dot{\varphi}(t-T_0);$$

$$\nabla_2\dot{\varphi}(t) = \nabla_1\dot{\varphi}(t) - \nabla_1\dot{\varphi}(t - T_0); \qquad (15)$$

where T_0 is the integration step, s. The time histories of these signals for t_m=6.2 s and t_m=7 s are shown in Fig. 5.

Figure 5

Graphs of the first and second order signal's inverse finite differences

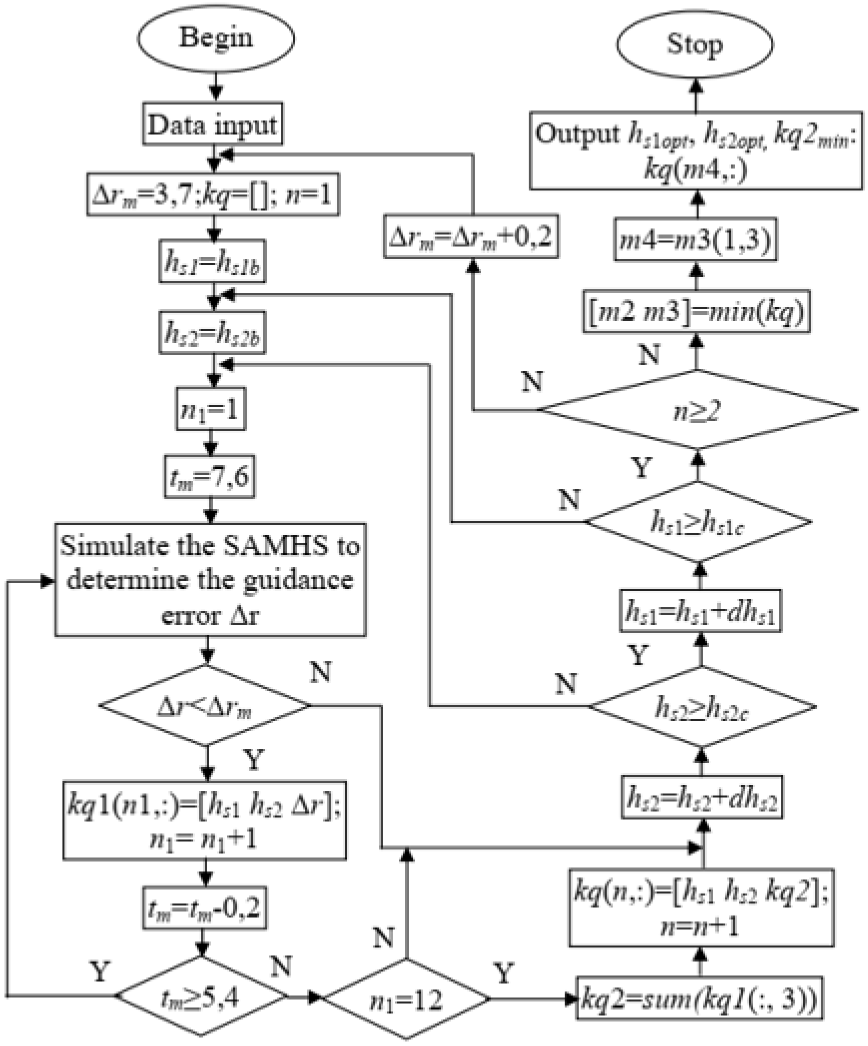


The signals $\nabla_1\dot{\varphi}_1$ and $\nabla_2\dot{\varphi}_1$ are used to construct the law governing the variation of the coefficient k_{pr}. It is proposed to apply the following law for its variation:

$$k_{pr} = \begin{cases} k_{propt},\ r \geq 3500; \\ \begin{cases} k_{pr},\ r \geq 800 \ \ and \ \ \nabla_1\dot{\varphi} \leq 5.10^{-5}; \\ \begin{cases} k_{pr},\ \ r \geq 800, \nabla_1\dot{\varphi} > 5.10^{-5},\ \nabla_2\dot{\varphi} \geq 45.10^{-8}; \\ k_{pr} + h_{s1opt}\nabla_2\dot{\varphi},\ r \geq 800, \nabla_1\dot{\varphi} > 5.10^{-5},\ \nabla_2\dot{\varphi} \geq 3.10^{-7}; \\ k_{pr} + h_{s2opt}\nabla_2\dot{\varphi},\ r \geq 800, \nabla_1\dot{\varphi} > 5.10^{-5},\ \nabla_2\dot{\varphi} < 3.10^{-7}; \end{cases} \end{cases} \\ \begin{cases} k_{pr} + h_{s2opt}\nabla_2\dot{\varphi},\ r < 800 \ \ and \ \ \nabla_1\dot{\varphi} > 0; \\ k_{pr}\ ,\ r < 800 \ \ and \ \ \nabla_1\dot{\varphi} < 0; \end{cases} \end{cases}$$

Figure 6

Algorithm for determining the optimal values of h_{s1opt}, h_{s2opt}



The coefficient k_{pr} is upper-bounded by $k_{pr} \leq 14.1$. The subsequent task is to determine the optimal values of the parameters h_{s1opt} and h_{s2opt}, which minimize the cumulative guidance error for t_m=5.4 s, t_m=5.6 s, …, t_m=7.6 s. The algorithm used to determine the optimal values of h_{s1opt} and h_{s2opt} is shown in Fig. 6.

Applying the proposed algorithm yields the optimal parameter values h_{s1opt}=109 and h_{s2opt}=334000, with a total guidance error of 12.55 m for t_m=5.4 s, t_m=5.6 s, …, t_m =7.6 s. This configuration is hereafter referred to as **System 2**. The corresponding guidance error Δr when engaging maneuvering targets is presented in Table 3.

Table 3

Guidance error system 2 when engaging maneuvering targets

Δx_0=15000 m; Δy_0=5000 m; v_t=800 m/s; Θ_t=180 deg; w_t=-70 m/s^2														
t_m, c	4	5	5,4	5,6	5,8	6	6,2	6,4	6,6	6,8	7	7,2	7,4	7,6
Δr, м	0,54	0,53	0,5	0,38	0,83	0,34	0,8	0,9	0,61	0,59	0,93	1,79	3,71	1,17

By comparing the guidance errors Δr in Tables 2 and 3, it can be observed that the proposed law for varying the coefficient k_{pr} significantly improves the accuracy of the surface-to-air missile homing system when engaging maneuvering targets.

4 REDUCTION OF THE INFLUENCE OF RANDOM DISTURBANCE ON SAMHS ACCURACY

It is assumed that the homing head operates under disturbance conditions. In this case, system (13) can be written in the following form:

$$\begin{cases} \frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}u + w; \\ y = \mathbf{c}\mathbf{x} + v; \end{cases} \qquad (15)$$

$$A = \begin{pmatrix} -\frac{1}{T_1} & 0 & -\frac{k_1}{T_1} \\ \frac{k_2}{HT_2} & -\frac{1}{T_2} & 0 \\ 0 & 1 & 0 \end{pmatrix}; \; B = \begin{pmatrix} \frac{k_1}{T_1} \\ 0 \\ 0 \end{pmatrix}; \; C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix};$$

where:

$x_1=u_{hh}$,

$x_2=u_{hh1}$,

and $x_3=\varphi_{fb}$;

$u=\varphi$;

w denotes the model disturbance,

v the measurement noise,

and y the output signal of the homing head.

It is assumed that the initial coordinate differences between the missile and the target are Δx_0=15000 m and Δy_0=5000 m. The target moves at a constant velocity of 800 m/s with a heading angle Θ_t=180∘. At time t_m=7 s after the start of homing, the target performs a maneuver with an acceleration of −70 m/s2. Simulation results show that, in the presence of disturbances, the guidance error of both System 1 and System 2 can reach up to 2500 m. To reduce the guidance error, a Kalman filter with identical parameters is applied to both systems.

The statistical simulation method described in [10] is employed. A confidence interval α_c=3, an allowable computation error α_{allow}=0.1, and an initial sample of 500 launches are selected. The required number of launches N_{rq}, the mean value M, the standard deviation σ, and the maximum value of guidance error Δr_{max} are presented in Table 4. The probability density functions of the guidance error for Systems 1 and 2 with the Kalman filter applied are shown in Fig. 7.

Table 4

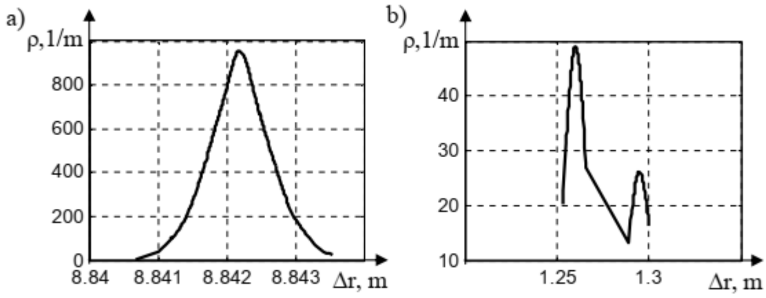
Error in guidance before and after applying the Kalman filter

System number	N_{rq}	M, m	σ, m	Δr_{max}, m
System 1	1	8.8422	0.004	8.8435
System 2	1	1.27	0.017	1.3

As shown in Table 4, the application of the Kalman filter significantly reduces guidance errors in the presence of disturbances. Moreover, the system employing the proposed adaptive law for varying the coefficient k_{pr} demonstrates higher accuracy than the system with a fixed k_{pr}.

Figure 7

Probability density of the system's guidance error without a Kalman filter (a) and with a Kalman filter (b)

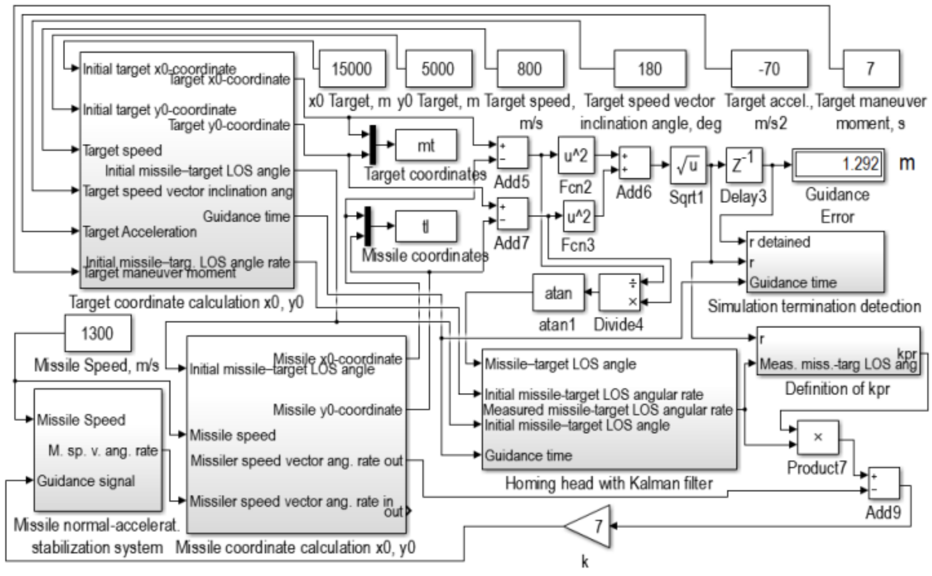


5 MODELING A MISSILE HOMING SYSTEM IN SIMULINK

The high-level Simulink model of the proposed surface-to-air missile homing system is shown in Fig. 8. It consists of six subsystems.

Figure 8

Modeling a missile homing system in Simulink



The subsystem **"Target coordinate calculation x_0, y_0"** implements the system of equations (11).

The subsystem **"Missile normal-acceleration stabilization system"** implements transfer functions (1)-(5), the actuator model, the angular rate sensor and accelerometer models in the form of the transfer function (7), and the first two equations of system (9). In addition, it includes *To Workspace* blocks labeled **“Control surface deflection angle”** and **“Angle of attack”** for storing the corresponding variables.

The subsystem **“Missile coordinate calculation x_0, y_0”** implements the system of equations (10).

The subsystem **“Gyro-stabilized homing head with Kalman filter”** includes a *Random Number* block for disturbance generation and implements the Kalman filter based on system (15).

The subsystem **“Simulation termination detection”** automatically determines the simulation end time.

In addition to the subsystems described above, the model includes blocks implementing system (12) and the third equation of system (9), as well as *To Workspace* blocks labeled **“Target coordinates”**, **“Missile coordinates”**, and **“Guidance time”** for data storage.

Prior to simulation, the following settings are specified: simulation stop time of 100 s; solver type *Fixed-step*; solver *ode4 (Runge–Kutta)*; and fixed step size of 0.001 s. In the *Random Number* block, the mean value is set to 0, the variance value, and the seed is assigned a random value, e.g., round(10000*normrnd(25,3)). As shown in Fig. 8, the guidance error at t_m=7 s is 1.292 m.

6 CONCLUSION

Based on the conducted study, the following conclusions can be drawn.

A linearized missile model can be effectively employed in the synthesis of a surface-to-air missile homing system using the proportional navigation method. The proposed synthesis approach enables the design of a high-accuracy missile homing system.

Deviations of the missile aerodynamic coefficients from their nominal values within ±20% lead to a noticeable increase in guidance error for short guidance times. As the guidance time increases-either due to a reduction in missile velocity within ±20% or an increase in the initial missile-target coordinata difference-the influence of aerodynamic coefficient variations on the guidance error decreases.

Adaptive variation of the proportional navigation gain according to the proposed law significantly improves the accuracy of the homing system. The application of this approach requires the availability of range information to the target.

Finally, the use of a Kalman filter in the homing head substantially reduces the impact of disturbances on guidance accuracy.

REFERENCES

Book: Golubov I.S., Svetlov V.G. *Design of Anti-Aircraft Guided Missiles. Second Edition*. Moscow, 2001. Pages 321-324.

Book: Kanashchenkova A.I., Merkulova V.I. *Aviation Radio-Control Systems. Vol. 2: Radio-Electronic Homing Systems*. Moscow: Radiotekhnika, 2003: 23–30.

Book: Pupkov K.A., Egupov N.D., Kolesnikov L.V. *High-Precision Homing Systems: Calculation and Design. Computational Experiment*. Moscow: FIZMATLIT, 2011: 49-51, 54, 295.

Book: Arkhagelsky I.I., Afanasev P.P., Boltov E.G., Galubev I.S., Matvienco A.M., Mizrokhi V.Y., Novikon V.N., Ostapenko S.N., Svetlov V.G. *Design of Anti-Aircraft Guided Missiles*. Moscow: MAI Publishing, 2001: 321-344.

Book: Timofeev N.N., Shestun A.N. *Design of Nonstationary Dynamic Control Systems for Aircraft.* St. Petersburg, BSTU, 2001: 9-43.

Journal Article: Do Quang Thong. *Synthesis of a High-Precision Missile Homing System with an Allowable Stability Margin of the Normal-Acceleration Stabilization System.* Mechatronics, Automation and Control. 2021. Vol. 22. No. 7: 365-373.

Journal Article: Do Quang Thong. *Synthesis of an Optimal Missile Homing System Using the Proportional Guidance Method*. Devices and Systems. Control, Monitoring, and Diagnostics. 8.2021. Pages 17-24.

Book: Lebedev A.A., Karabanov V.A. *Dynamics of Unmanned Aerial Vehicle Control Systems*. Moscow, Mashinostroenie, 1965: 156, 158, 249, 410-513.

Journal Article: Do Quang Thong. *Synthesis of High-Precision Missile Homing System Using Proportional Guidance Method*. Mechatronics, automation and control. 2020. Volume 21. No. 4. P. 242-248.

Book: Emelyanov V.Yu.. *Methods of modeling stochastic control systems*. BSTU Voenmekh. Saint Petersburg. 1997. P.40-50, 61, 62.

Authors' Contribution

All authors contributed equally to the development of this article.

Data availability

All datasets relevant to this study's findings are fully available within the article.

How to cite this article (APA)

Tuan, N. N., & Thong, D. Q. INCREASING THE ACCURACY OF THE ANTI-AIRCRAFT MISSILE HOMING SYSTEM BY CHANGING THE PROPORTIONALTY COEFFICIENT OF THE HOMING LAW. *Veredas Do Direito*, e234471. https://doi.org/10.18623/rvd.v23.n4.4471